

## Axioms of Addition and Multiplication over $\mathfrak{R}$

$a, b, c \in \mathfrak{R}$ ;  $a, b, c$  are Real Numbers

$a + b$ is a unique real number. Ex. The sum of any two real numbers only gives one real answer.	<b>Closure Axiom for Addition</b>
$(a + b) + c = a + (b + c)$ Ex. $(2 + 3) + 4 = 2 + (3 + 4)$	<b>Associative Axiom for Addition</b>
$a + b = b + a$	<b>Commutative Axiom for Addition</b>
There exists an element $0 \in \mathfrak{R}$ such that for each $a \in \mathfrak{R}$ , $0 + a = a$ and $a + 0 = a$ . Ex. $4 + 0 = 4$ , $0 + 4 = 4$	<b>Identity Axiom for Addition</b>
There exists an element $-a \in \mathfrak{R}$ for each $a \in \mathfrak{R}$ , such that $a + (-a) = 0$ and $-a + a = 0$ . Ex. Every real number has an opposite that when added, 0 is the answer. $4 + (-4) = 0$	<b>Axiom of Additive Inverses</b>
$ab$ is a unique real number. Ex. The product of any two real number only gives one real answer.	<b>Closure Axiom for Multiplication</b>
$(ab)c = a(bc)$ Ex. $(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$	<b>Associative Axiom for Multiplication</b>
$ab = ba$ Ex. $2 \cdot 3 = 3 \cdot 2$	<b>Commutative Axiom for Multiplication</b>
There exists an element $1 \in \mathfrak{R}$ such that for each $a \in \mathfrak{R}$ , $1 \cdot a = a$ and $a \cdot 1 = a$ . Ex. $4 \cdot 1 = 4$ , $1 \cdot 4 = 4$	<b>Identity Axiom for Multiplication</b>
There exists an element $\frac{1}{a} \in \mathfrak{R}$ for each nonzero $a \in \mathfrak{R}$ , such that $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$ . Ex. Every nonzero real number has a reciprocal that when multiplied, 1 is the answer. $\frac{4}{1} \cdot \frac{1}{4} = 1$	<b>Axiom for Multiplicative Inverses</b>
$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ Ex. $2(3 + 4) = 2 \cdot 3 + 2 \cdot 4$ and $(3 + 4)2 = 3 \cdot 2 + 4 \cdot 2$	<b>Distributive Axiom</b>

## Axioms of Equality for $\mathfrak{R}$

$a, b, c \in \mathfrak{R}$ ;  $a, b, c$  are Real Numbers

$a = a$	<b>Reflexive Property</b>
If $a = b$ , then $b = a$ .	<b>Symmetric Property</b>
If $a = b$ and $b = c$ , then $a = c$ .	<b>Transitive Property</b>

Use the following tables to answer the given questions: Justify each answer.

$\sigma$	0	2	4	6
0	4	0	6	4
2	0	2	8	6
4	6	4	2	0
6	2	6	0	4

Is  $\sigma$  closed?

$\lambda$	0	2	4	6
0	4	0	6	2
2	0	2	4	6
4	6	4	2	0
6	2	6	0	4

Is  $\lambda$  closed?

Is  $\sigma$  commutative?

Is  $\lambda$  commutative?

Is  $\sigma$  associative?

Is  $\lambda$  associative?

Do inverse elements exist for each element  $\{0,2,4,6\}$  under the operation of  $\sigma$  ?

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Does an identity element exist under the binary operation  $\sigma$  ?

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